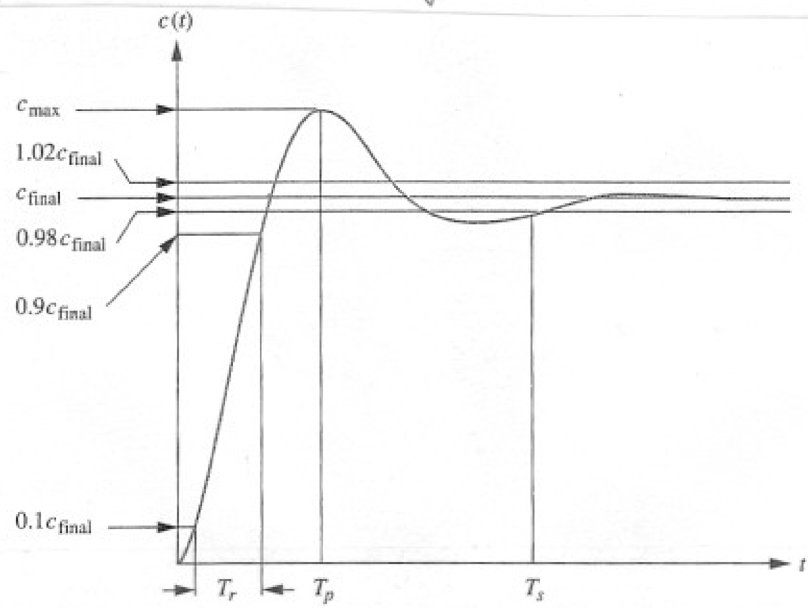
Second Order Systems cont.

Most second order systems are underdamped ($\zeta < 1$) The step response of an underdamped 2nd order system has a typical plot as shown below.



There are several parameters that characterize the response of the system

Rise Time - The time required for the waveform to go from 0.1 to 0.9 of the final value.

$$t_r\Big|_{10\%}^{90\%} \approx \frac{1 + 1.1\zeta + 1.4\zeta^2}{\omega_n} \qquad t_r\Big|_{0}^{100\%} = \frac{1}{\omega_n\sqrt{1-\zeta^2}} \tan^{-1}\left\{\frac{\sqrt{1-\zeta^2}}{\zeta}\right\}$$

Maximum Overshoot - The amount the output exceeds the final value which occurs at the peak time

$$OS_{MAX} = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$c_{max} = 1 + OS_{max} \quad \text{(step response)}$$

Percent Overshoot - The amount that the waveform overshoots the steady-state or final value at the peak time, expressed as a percentage of the steady state value.

$$\%OS = 100\left\{\frac{C_{max} - C_{final}}{C_{final}}\right\} = 100\, e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$$

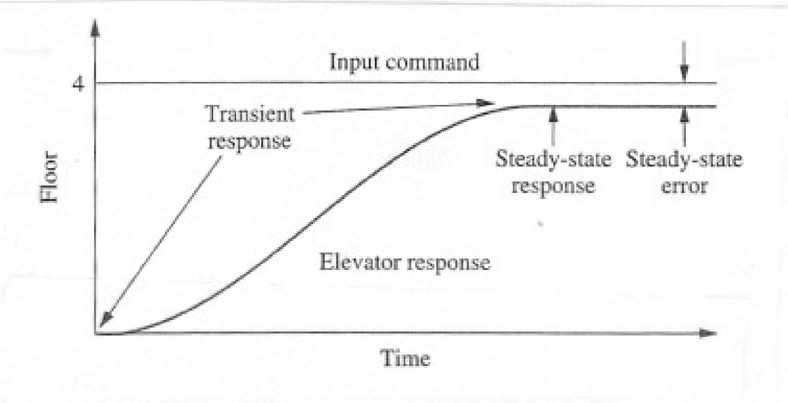
Settling Time - The time required for the transient's damped oscillations to reach and stay within a percentage of the steady-state value (typically 2%).

$$t_s = \underbrace{4\tau}_{\text{first order system}} \Big| = \underbrace{\frac{4}{\zeta\omega_n}}_{\text{second order system}} \Big| \qquad (2\%)$$

Peak Time - The time required to reach the first or maximum peak.

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

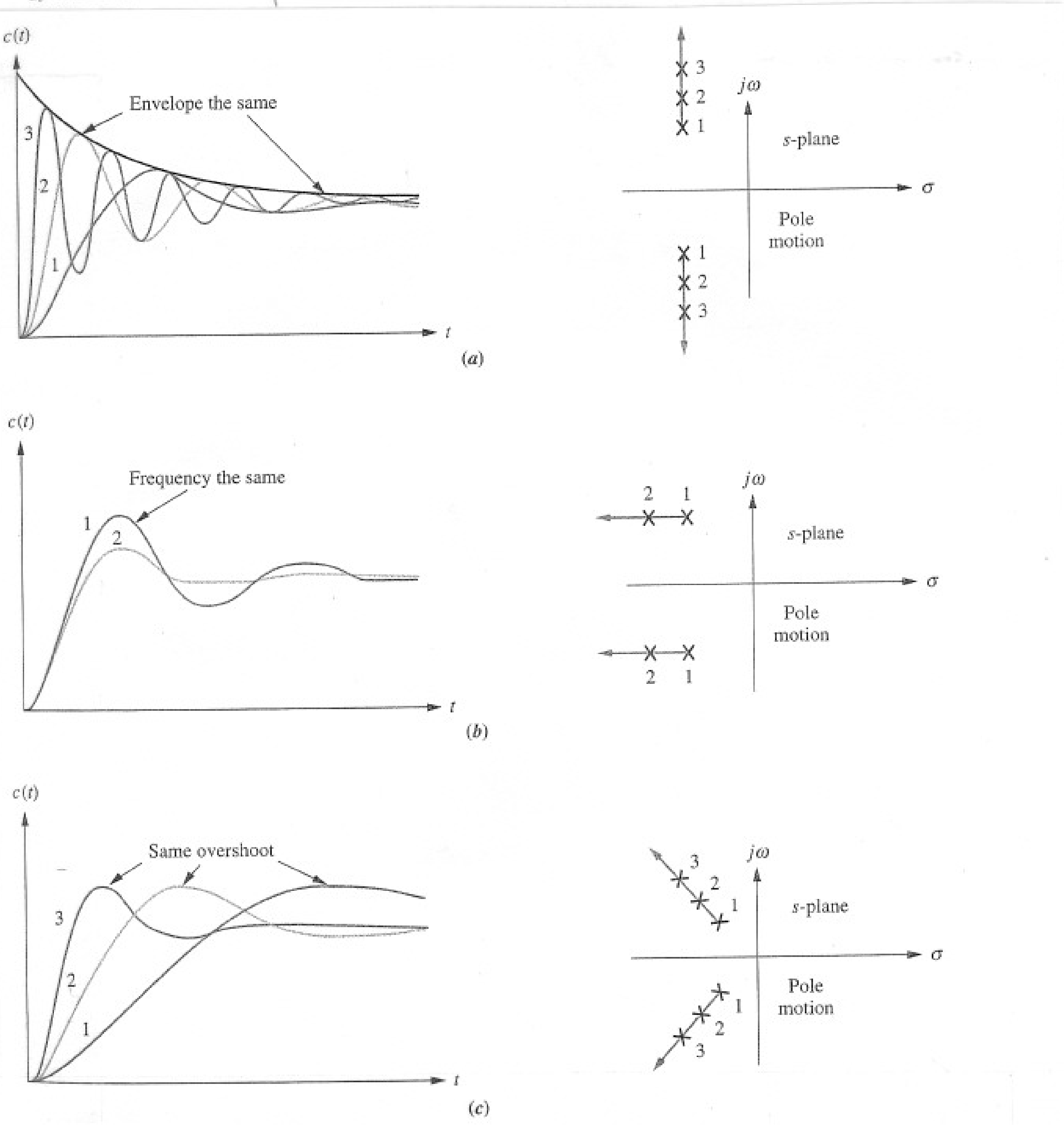
Steady State Error - The difference between the desired and the actual output of the system.



So far we have learned about first order and second order systems. input $\longrightarrow \boxed{G(s)} \longrightarrow$ output

$$G(s) = \frac{1}{\tau s + 1} \quad ; \quad G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Depending on τ, ω_n, and ζ the systems respond differently to the input. These three parameters ultimately affect the pole locations that affect the system response.



- Now we would like to alter the response of the system such that it behaves with improved performance. We can do this applying feedback to the original system (plant). By applying feedback we can change parameters such as rise time, percent overshoot, steady state error, settling time, etc. (see 1-5, 3-1, 3-2)
- Prior to learning about feedback control systems, let's review block diagram algebra

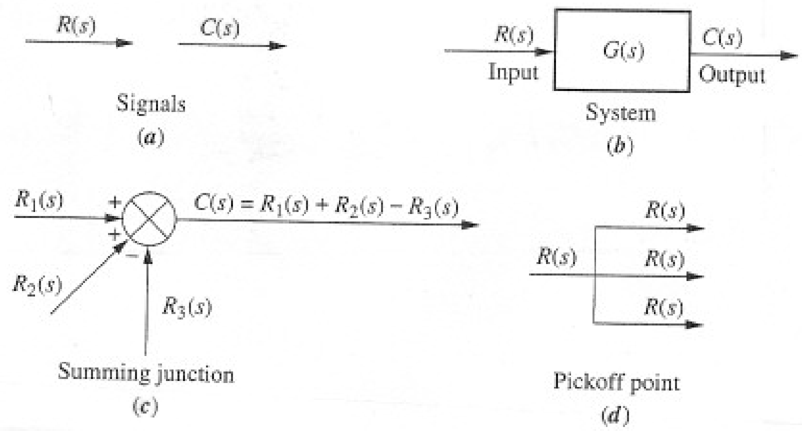


Figure 5.2
Components of a block diagram for a linear, time-invariant system

Figure 5.3
a. Cascaded subsystems;
b. equivalent transfer function

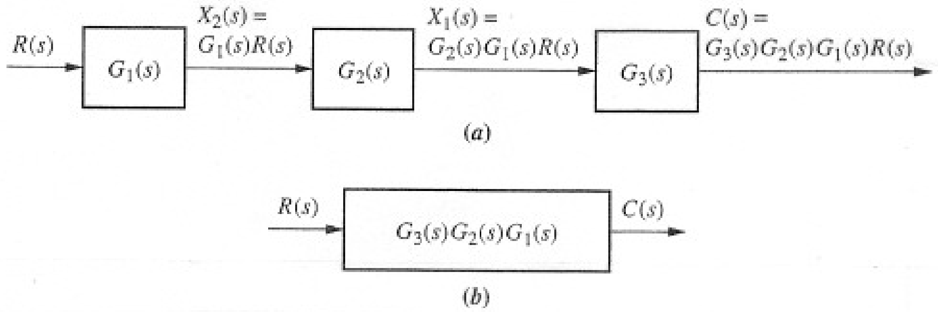
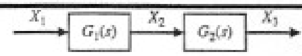
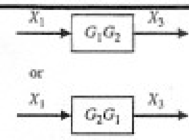
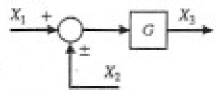
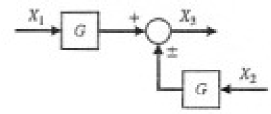
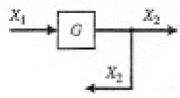
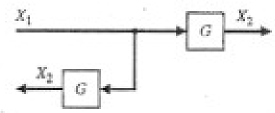
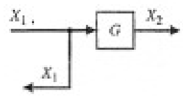
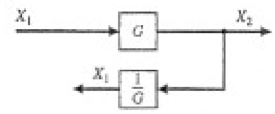
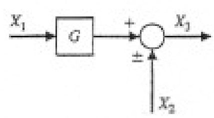
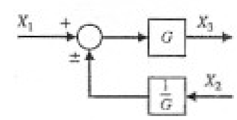
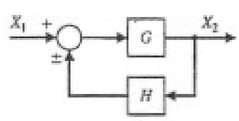


Table 1. Block Diagram Transformations[1]

Transformation	Original Diagram	Equivalent Diagram
1. Combining blocks in cascade		
2. Moving a summing point behind a block		
3. Moving a pickoff point ahead of a block		
4. Moving a pickoff point behind a block		
5. Moving a summing point ahead of a block		
6. Eliminating a feedback loop		

- Let's suppose we have a dynamic system, $G_p(s)$, and refer to it as the "plant". For this open-loop system the response is independent of its output and only depends on its input.

- If the performance of the system is not sufficient it can usually be improved by making the input dependent on the output via a feedback signal
- The most general control system has the following form